

Year 12 Methods Units 3,4 Test 3 2020

Section 1 Calculator Free Discrete Random Variables

STUDENT'S NAME

DATE: Thursday 14th May

TIME: 15 minutes

MARKS: 18

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (7 marks)

(a) Determine if each of the following are probability functions, justify your answer either way.

| (i) | | | | | | [2] |
|-----|----------|----|-----|-----|-----|-----|
| | X | -1 | 1 | 2 | 3 | |
| | P(X = x) | 0 | 0.3 | 0.4 | 0.3 | |

(ii)
$$P(X = x) = \frac{x^2}{30}$$
; $x = 1, 2, 3, 4$ [3]

(b) An experiment is conducted where a ball is randomly picked from a bag containing red and green balls. What condition(s) must be placed on the experiment for it to be considered a Bernoulli experiment? [2]

2. (11 marks)

The discrete random variable *X* has a cumulative probability distribution given by:

| x | -1 | 0 | 1 | 2 | 3 |
|--------------|-----|---------|---------|----------|----------|
| $P(X \le x)$ | 0.2 | 0.2 + a | 0.3 + a | 0.3 + 2a | 0.5 + 2a |

(a) Determine *a* using the distribution above.

(b) Calculate:

(i)
$$P(X < 2)$$
 [1]

- (ii) $P(X > -1 | X \le 2)$ [2]
- (b) Determine E(X) [2]

(c) If
$$E(X^2) = \frac{31}{10}$$
, determine Var (X). [2]

The random variable Y = 6 - 2X

- (d) (i) Determine E(Y) [1]
 - (ii) Determine Var(Y) [1]

[2]



Year 12 Methods Units 3,4 Test 3 2020

Section 2 Calculator Assumed Discrete Random Variables

STUDENT'S NAME

DATE: Thursday 14th May

TIME: 35 minutes

MARKS: 38

INSTRUCTIONS:

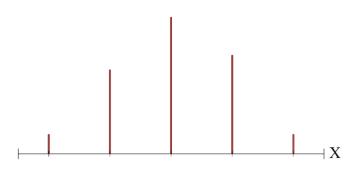
Standard Items:Pens, pencils, drawing templates, eraserSpecial Items:Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

4. (3 marks)

This graph represents a binomial probability distribution.

The height of the last column is 0.053



(a) State the value of n.

[1]

(b) State the probability of success for this binomial distribution, correct to 2 decimal places. [2]

5. (8 marks)

In Australia, approximately 9% of people have the blood type O-negative. On any given day the Red Cross needs to collect blood from at least 2 donors with O-negative blood.

On Wednesday they collect blood from 20 donors of various blood types, let the random variable X be the number of donors with O-negative blood.

- (a) State the probability distribution for X.
- (b) Calculate the probability that the target of at least 2 donors with O-negative blood will be met. [2]

(c) Given that at least 15 donors without O-negative blood have been found from the 20, what is the probability that there will be exactly 4 donors with O-negative blood? [3]

(d) How many donors would be needed to give at least a 99% chance that at least 1 donor with blood type O-negative is found? [2]

[1]

6. (4 marks)

A school has analysed the examination scores for all its Year 12 students taking Methods as a subject. Let X: the examination percentage scores of all the Methods Year 12 students at the school. The school found that the mean was 75 with a standard deviation of 22.

The school has decided to scale the results using the transformation Y = aX + b where *a* and *b* are constants and *Y*: the scaled percentage scores. The aim is to change the mean to 60 and the standard deviation to 15.

Determine the values of a and b.

7. (8 marks)

Amal noticed that in Methods, which he attends four days a week, the chance of being set homework on any one day was 70% and independent of the previous day.

(a) Determine the probability that Amal is set homework in this class;

| (i) | the next time he attends but not the lesson after | [1] |
|-----|---|-----|
|-----|---|-----|

- (ii) exactly twice in the next week [2]
- (b) Determine the probability that over a ten-week term, Amal will be given homework twice a week in Methods, at least 7 times. [2]

(c) Determine that probability that Amal would receive his fourth lot of homework on his seventh lesson of Methods. [3]

8. (5 marks)

A discrete random variable X has the following probability distribution:

| | x | 0 | 1 | 2 | 3 |
|-----|--------------|-------|-------|---|---|
| P(X | = <i>x</i>) | 0.064 | 0.288 | У | z |

Note: for all other x, P(X = x) = 0

- (a) Determine the value of y in terms of z. [1]
- (b) Given X is a binomial random variable, determine the values of y and z. [4]

9. (10 marks)

For Edmund Rice Day, one PCG will have a game involving rolling two regular six-sided dice. The **difference** in the values on the two dice will be the score. The game will cost \$2 for an attempt with a payout back to the customer of \$8 if the score is 5, a payout of \$5 if the score is 2 and they get their money back if the score is 4. All other scores do not win a prize

(a) Indicate the possible outcomes for one attempt of rolling two dice. [2]

Let X be the **profit** made by the PCG on one attempt.

(b) Complete the table below

[2]

| x | | |
|-------------------|--|--|
| $\mathbf{P}(X=x)$ | | |

The PCG is anticipating that there will be 600 attempts of the game on the day.

(c) How much total profit is the PCG expecting?

[2]

The PCG decide this is not enough profit and decides to increase the cost of one attempt so that the expected profit on the 600 rolls would be at least \$400.

(d) Noting that all transactions are made with cash, what is the **minimum** charge to **ensure** expected profit is at least \$400? [4]